

**Solution to (1)**

Answer: (A)

The put-call parity formula (for a European call and a European put on a stock with the same strike price and maturity date) is

$$\begin{aligned}C - P &= F_{0,T}^P(S) - F_{0,T}^P(K) \\ &= F_{0,T}^P(S) - \text{PV}_{0,T}(K) \\ &= F_{0,T}^P(S) - Ke^{-rT} \\ &= S_0 - Ke^{-rT},\end{aligned}$$

because the stock pays no dividends

We are given that  $C - P = 0.15$ ,  $S_0 = 60$ ,  $K = 70$  and  $T = 4$ . Then,  $r = 0.039$ .

**Remark 1:** If the stock pays  $n$  dividends of fixed amounts  $D_1, D_2, \dots, D_n$  at fixed times  $t_1, t_2, \dots, t_n$  prior to the option maturity date,  $T$ , then the put-call parity formula for European put and call options is

$$\begin{aligned}C - P &= F_{0,T}^P(S) - Ke^{-rT} \\ &= S_0 - \text{PV}_{0,T}(\text{Div}) - Ke^{-rT},\end{aligned}$$

where  $\text{PV}_{0,T}(\text{Div}) = \sum_{i=1}^n D_i e^{-rt_i}$  is the present value of all dividends up to time  $T$ . The

difference,  $S_0 - \text{PV}_{0,T}(\text{Div})$ , is the *prepaid forward price*  $F_{0,T}^P(S)$ .

**Remark 2:** The put-call parity formula above does not hold for American put and call options. For the American case, the parity relationship becomes

$$S_0 - \text{PV}_{0,T}(\text{Div}) - K \leq C - P \leq S_0 - Ke^{-rT}.$$

This result is given in Appendix 9A of McDonald (2006) but is not required for Exam MFE/3F. Nevertheless, you may want to try proving the inequalities as follows:

For the first inequality, consider a portfolio consisting of a European call plus an amount of cash equal to  $\text{PV}_{0,T}(\text{Div}) + K$ .

For the second inequality, consider a portfolio of an American put option plus one share of the stock.