Solution to (2) Answer: (D)

The prices are not arbitrage-free. To show that Mary's portfolio yields arbitrage profit, we follow the analysis in Table 9.7 on page 302 of McDonald (2006).

	Time 0	Time T			
		$S_T < 40$	$40 \le S_T < 50$	$50 \le S_T < 55$	$S_T \ge 55$
Buy 1 call Strike 40	– 11	0	$S_T - 40$	$S_T - 40$	$S_T - 40$
Sell 3 calls Strike 50	+ 18	0	0	$-3(S_T - 50)$	$-3(S_T - 50)$
Lend \$1	– 1	e^{rT}	e^{rT}	e^{rT}	e^{rT}
Buy 2 calls strike 55	– 6	0	0	0	$2(S_T - 55)$
Total	0	$e^{rT} > 0$	$e^{rT} + S_T - 40 > 0$	$e^{rT} + 2(55 - S_T) > 0$	$e^{rT} > 0$

Peter's portfolio makes arbitrage profit, because:

	Time-0 cash flow	Time-T cash flow
Buy 2 calls & sells 2 puts Strike 55	$2(-3 + 11) = 16$	$2(S_T - 55)$
Buy 1 call & sell 1 put Strike 40	$-11 + 3 = -8$	$S_T - 40$
Lend \$2	-2	$2e^{rT}$
Sell 3 calls & buy 3 puts Strike 50	$3(6 - 8) = -6$	$3(50 - S_T)$
Total	0	$2e^{rT}$

Remarks: Note that Mary's portfolio has no put options. The call option prices are not arbitrage-free; they do not satisfy the convexity condition (9.17) on page 300 of McDonald (2006). The time-T cash flow column in Peter's portfolio is due to the identity

$$\max[0, S - K] - \max[0, K - S] = S - K$$

(see also page 44).

In *Loss Models*, the textbook for Exam C/4, $\max[0, \alpha]$ is denoted as α_+. It appears in the context of stop-loss insurance, $(S - d)_+$, with S being the claim random variable and d the deductible. The identity above is a particular case of

$$x = x_+ - (-x)_+,$$

which says that every number is the difference between its positive part and negative part.