Solution to (3)

The payoff at the contract maturity date is

$$\pi \times (1 - y\%)\times\text{Max}[S(T)/S(0), (1 + g\%)^T]$$
$$= \pi \times (1 - y\%)\times\text{Max}[S(1)/S(0), (1 + g\%)^1] \quad \text{because } T = 1$$
$$= [\pi/S(0)](1 - y\%)\text{Max}[S(1), S(0)(1 + g\%)]$$
$$= (\pi/100)(1 - y\%)\text{Max}[S(1), 103] \quad \text{because } g = 3 \text{ \& } S(0)=100$$
$$= (\pi/100)(1 - y\%)\{S(1) + \text{Max}[0, 103 - S(1)]\}.$$

Now, Max[0, 103 – $S(1)$] is the payoff of a one-year European put option, with strike price \$103, on the stock index; the time-0 price of this option is given to be is \$15.21. Dividends are incorporated in the stock index (i.e., $\delta = 0$); therefore, $S(0)$ is the time-0 price for a time-1 payoff of amount $S(1)$. Because of the no-arbitrage principle, the time-0 price of the contract must be

$$(\pi/100)(1 - y\%)\{S(0) + 15.21\}$$
$$= (\pi/100)(1 - y\%) \times 115.21.$$

Therefore, the "break-even" equation is

$$\pi = (\pi/100)(1 - y\%)\times 115.21,$$

or

$$y\% = 100 \times (1 - 1/1.1521)\% = 13.202\%$$

Remarks:

(i) Many stock indexes, such as S&P500, do not incorporate dividend reinvestments. In such cases, the time-0 cost for receiving $S(1)$ at time 1 is the prepaid forward price $F_{0,1}^P(S)$, which is less than $S(0)$.

(ii) The identities

$$\text{Max}[S(T), K] = K + \text{Max}[S(T) - K, 0] = K + (S(T) - K)_+$$

and

$$\text{Max}[S(T), K] = S(T) + \text{Max}[0, K - S(T)] = S(T) + (K - S(T))_+$$

can lead to a derivation of the put-call parity formula. Such identities are useful for understanding Section 14.6 *Exchange Options* in McDonald (2006).