Solution to (4) Answer: (C)

First, we construct the two-period binomial tree for the stock price.

Year 0   Year 1   Year 2

```
                              32.9731
                  25.680
      20                      22.1028
                  17.214
                              14.8161
```
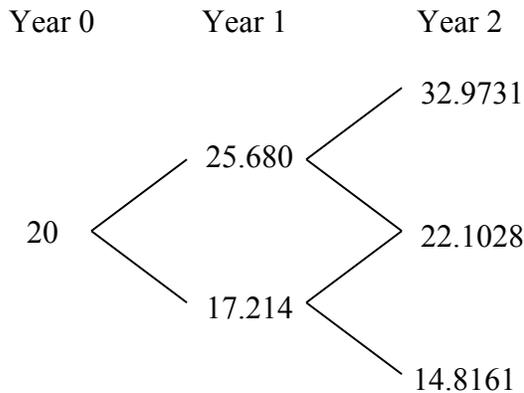
The calculations for the stock prices at various nodes are as follows:

$S_u = 20 \times 1.2840 = 25.680$
$S_d = 20 \times 0.8607 = 17.214$
$S_{uu} = 25.68 \times 1.2840 = 32.9731$
$S_{ud} = S_{du} = 17.214 \times 1.2840 = 22.1028$
$S_{dd} = 17.214 \times 0.8607 = 14.8161$

The risk-neutral probability for the stock price to go up is
$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.05} - 0.8607}{1.2840 - 0.8607} = 0.4502.$$
Thus, the risk-neutral probability for the stock price to go down is 0.5498.

If the option is exercised at time 2, the value of the call would be
$C_{uu} = (32.9731 - 22)_+ = 10.9731$
$C_{ud} = (22.1028 - 22)_+ = 0.1028$
$C_{dd} = (14.8161 - 22)_+ = 0$

If the option is European, then $C_u = e^{-0.05}[0.4502C_{uu} + 0.5498C_{ud}] = 4.7530$ and
$C_d = e^{-0.05}[0.4502C_{ud} + 0.5498C_{dd}] = 0.0440$.
But since the option is American, we should compare C_u and C_d with the value of the option if it is exercised at time 1, which is 3.68 and 0, respectively. Since 3.68 < 4.7530 and 0 < 0.0440, it is not optimal to exercise the option at time 1 whether the stock is in the up or down state. Thus the value of the option at time 1 is either 4.7530 or 0.0440.

Finally, the value of the call is
$C = e^{-0.05}[0.4502(4.7530) + 0.5498(0.0440)] = 2.0585$.

Remark: Since the stock pays no dividends, the price of an American call is the same as that of a European call. See pages 294-295 of McDonald (2006). The European option price can be calculated using the binomial probability formula. See formula (11.17) on page 358 and formula (19.1) on page 618 of McDonald (2006). The option price is

$$e^{-r(2h)}\left[\binom{2}{2}p*^2 C_{uu} + \binom{2}{1}p*(1-p*)C_{ud} + \binom{2}{0}(1-p*)^2 C_{dd}\right]$$

$= e^{-0.1}[(0.4502)^2 \times 10.9731 + 2 \times 0.4502 \times 0.5498 \times 0.1028 + 0]$

$= 2.0507$