Solution to (7)

Let $X(t)$ be the exchange rate of U.S. dollar per Japanese yen at time t. That is, at time t,

$$¥1 = \$X(t).$$

We are given that $X(0) = 1/120$.

At time ¼, Company A will receive ¥ 120 billion, which is exchanged to \$[120 billion × X(¼)]. However, Company A would like to have

$$\$ \text{ Max}[1 \text{ billion}, \ 120 \text{ billion} \times X(¼)],$$

which can be decomposed as

$$\$120 \text{ billion} \times X(¼) \ + \ \$ \text{ Max}[1 \text{ billion} \ - \ 120 \text{ billion} \times X(¼), 0],$$

or

$$\$120 \text{ billion} \times \{X(¼) \ + \ \text{Max}[120^{-1} - \ X(¼), 0]\}.$$

Thus, Company A purchases 120 billion units of a put option whose payoff three months from now is

$$\$ \text{ Max}[120^{-1} - \ X(¼), 0].$$

The exchange rate can be viewed as the price, in US dollar, of a traded asset, which is the Japanese yen. The continuously compounded risk-free interest rate in Japan can be interpreted as δ, the dividend yield of the asset. See also page 381 of McDonald (2006) for the *Garman-Kohlhagen model*. Then, we have

$$r = 0.035, \delta = 0.015, S = X(0) = 1/120, K = 1/120, T = ¼.$$

Because the logarithm of the exchange rate of yen per dollar is an arithmetic Brownian motion, its negative, which is the logarithm of the exchange rate of dollar per yen, is also an arithmetic Brownian motion and has the SAME volatility. Therefore, $\{X(t)\}$ is a geometric Brownian motion, and the put option can be priced using the Black-Scholes formula for European put options. It remains to determine the value of σ, which is given by the equation

$$\sigma\sqrt{\frac{1}{365}} \ = \ 0.261712 \ \%.$$

Hence,

$$\sigma \ = \ 0.05.$$

Therefore,

$$d_1 \ = \ \frac{(r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}} \ = \ \frac{(0.035 - 0.015 + 0.05^2/2)/4}{0.05\sqrt{1/4}} \ = \ 0.2125$$

and

$$d_2 \ = \ d_1 \ - \ \sigma\sqrt{T} \ = \ 0.2125 \ - \ 0.05/2 \ = \ 0.1875.$$

By (12.3) of McDonald (2006), the time-0 price of 120 billion units of the put option is

$$\$120 \text{ billion} \times [Ke^{-rT}N(-d_2) \ - \ X(0)e^{-\delta T}N(-d_1)]$$

$$= \ \$ \ [e^{-rT}N(-d_2) \ - \ e^{-\delta T}N(-d_1)] \text{ billion} \qquad \text{because } K = X(0) = 1/120$$

Using the Cumulative Normal Distribution Calculator, we obtain $N(-0.1875) = 0.42563$ and $N(-0.2125) = 0.41586$.

Thus, Company A's option cost is

$$e^{-0.035/4}\times 0.42563 \; - \; e^{-0.015/4}\times 0.41586$$
$$= \; 0.007618538 \text{ billion}$$
$$\approx \; 7.62 \text{ million.}$$

Remarks:

(i) Suppose that the problem is to be solved using options on the exchange rate of Japanese yen per US dollar, i.e., using yen-denominated options. Let

$$\$1 \; = \; ¥U(t)$$

at time t, i.e., $U(t) = 1/X(t)$.

Because Company A is worried that the dollar may increase in value with respect to the yen, it buys 1 billion units of a 3-month yen-denominated European *call* option, with exercise price ¥120. The payoff of the option at time ¼ is

$$¥ \text{ Max}[U(¼) \; - \; 120, 0].$$

To apply the Black-Scholes call option formula (12.1) to determine the time-0 price in yen, use

$r = 0.015$, $\delta = 0.035$, $S = U(0) = 120$, $K = 120$, $T = ¼$, and $\sigma = 0.05$.

Then, divide this price by 120 to get the time-0 option price in dollars. We get the same price as above, because d_1 here is $-d_2$ of above.

The above is a special case of formula (9.7) on page 292 of McDonald (2006).

(ii) There is a cheaper solution for Company A. At time 0, borrow

$$¥ \; 120\times\exp(- \; ¼ \; r_{¥}) \text{ billion,}$$

and immediately convert this amount to US dollars. The loan is repaid with interest at time ¼ when the deal is closed.
On the other hand, with the option purchase, Company A will benefit if the yen increases in value with respect to the dollar.