Solution to (17) Answer (A)

This problem is based on Sections 11.3 and 11.4 of McDonald (2006), in particular, Table 11.1 on page 361.

Let $\{r_j\}$ denote the continuously compounded monthly returns. Thus, $r_1 = \ln(80/64)$, $r_2 = \ln(64/80)$, $r_3 = \ln(80/64)$, $r_4 = \ln(64/80)$, $r_5 = \ln(80/100)$, $r_6 = \ln(100/80)$, $r_7 = \ln(80/64)$, and $r_8 = \ln(64/80)$. Note that four of them are ln(1.25) and the other four are –ln(1.25); in particular, their mean is zero.

The (unbiased) sample variance of the non-annualized monthly returns is

$$\frac{1}{n-1}\sum_{j=1}^{n}(r_j - \bar{r})^2 = \frac{1}{7}\sum_{j=1}^{8}(r_j - \bar{r})^2 = \frac{1}{7}\sum_{j=1}^{8}(r_j)^2 = \frac{8}{7}[\ln(1.25)]^2.$$

The annual standard deviation is related to the monthly standard deviation by formula (11.5),

$$\sigma = \frac{\sigma_h}{\sqrt{h}},$$

where $h = 1/12$. Thus, the historical volatility is

$$\sqrt{12} \times \sqrt{\frac{8}{7}} \times \ln(1.25) = 82.6\%.$$

Remarks: Further discussion is given in Section 23.2 of McDonald (2006). Suppose that we observe n continuously compounded returns over the time period $[\tau, \tau + T]$. Then, $h = T/n$, and the historical annual variance of returns is estimated as

$$\frac{1}{h}\frac{1}{n-1}\sum_{j=1}^{n}(r_j - \bar{r})^2 = \frac{1}{T}\frac{n}{n-1}\sum_{j=1}^{n}(r_j - \bar{r})^2 .$$

Now,

$$\bar{r} = \frac{1}{n}\sum_{j=1}^{n} r_j = \frac{1}{n}\ln\frac{S(\tau + T)}{S(\tau)},$$

which is close to zero when n is large. Thus, a simpler estimation formula is $\frac{1}{h}\frac{1}{n-1}\sum_{j=1}^{n}(r_j)^2$ which is formula (23.2) on page 744, or equivalently, $\frac{1}{T}\frac{n}{n-1}\sum_{j=1}^{n}(r_j)^2$ which is the formula in footnote 9 on page 756. The last formula is related to #10 in this set of sample problems: With probability 1,

$$\lim_{n\to\infty}\sum_{j=1}^{n}[\ln S(jT/n) - \ln S((j-1)T/n)]^2 = \sigma^2 T.$$