Solution to (19) Answer: (C)

This problem is based on Exercise 14.21 on page 465 of McDonald (2006).

Let S_1 denote the stock price at the end of one year. Apply the Black-Scholes formula to calculate the price of the at-the-money call one year from today, conditioning on S_1.

$d_1 = [\ln(S_1/S_1) + (r + \sigma^2/2)T]/(\sigma\sqrt{T}) = (r + \sigma^2/2)/\sigma = 0.41667$, which turns out to be independent of S_1.

$d_2 = d_1 - \sigma\sqrt{T} = d_1 - \sigma = 0.11667$

The value of the forward start option at time 1 is

$$\begin{aligned} C(S_1) &= S_1N(d_1) - S_1e^{-r}N(d_2) \\ &= S_1[N(0.41667) - e^{-0.08}N(0.11667)] \\ &= S_1[0.66154 - e^{-0.08}\times 0.54644] \\ &= 0.157112S_1. \end{aligned}$$

(Note that, when viewed from time 0, S_1 is a random variable.)

Thus, the time-0 price of the forward start option must be 0.157112 multiplied by the time-0 price of a security that gives S_1 as payoff at time 1, i.e., multiplied by the prepaid forward price $F_{0,1}^P(S)$. Hence, the time-0 price of the forward start option is

$$0.157112\times F_{0,1}^P(S) = 0.157112\times e^{-0.08}\times F_{0,1}(S) = 0.157112\times e^{-0.08}\times 100 = 14.5033$$

Remark: A key to pricing the forward start option is that d_1 and d_2 turn out to be independent of the stock price. This is the case if the strike price of the call option will be set as a fixed percentage of the stock price at the issue date of the call option.