Solution to (26) Answer: (D)

$T = \frac{1}{2};\quad \text{PV}_{0,T}(K) = Ke^{-rT} = 100e^{-0.1/2} = 100e^{-0.05} = 95.1229 \approx 95.12.$

By (9.9) on page 293 of McDonald (2006), we have

$$S(0) \ge C_{Am} \ge C_{Eu} \ge \text{Max}[0,\ F_{0,T}^{P}(S) - \text{PV}_{0,T}(K)].$$

Because the stock pays no dividends, the above becomes

$$S(0) \ge C_{Am} = C_{Eu} \ge \text{Max}[0,\ S(0) - \text{PV}_{0,T}(K)].$$

Thus, the shaded region in II contains C_{Am} and C_{Eu}. (The shaded region in I also does, but it is a larger region.)

By (9.10) on page 294 of McDonald (2006), we have

$$K \ge P_{Am} \ge P_{Eu} \ge \text{Max}[0, \text{PV}_{0,T}(K) - F_{0,T}^{P}(S)]$$
$$= \text{Max}[0, \text{PV}_{0,T}(K) - S(0)]$$

because the stock pays no dividends. However, the region bounded above by $\pi = K$ and bounded below by $\pi = \text{Max}[0, \text{PV}_{0,T}(K) - S]$ is not given by III or IV.

Because an American option can be exercised immediately, we have a tighter lower bound for an American put,

$$P_{Am} \ge \text{Max}[0,\ K - S(0)].$$

Thus,

$$K \ge P_{Am} \ge \text{Max}[0,\ K - S(0)],$$

showing that the shaded region in III contains P_{Am}.

For a European put, we can use put-call parity and the inequality $S(0) \ge C_{Eu}$ to get a tighter upper bound,

$$\text{PV}_{0,T}(K) \ge P_{Eu}.$$

Thus,

$$\text{PV}_{0,T}(K) \ge P_{Eu} \ge \text{Max}[0,\ \text{PV}_{0,T}(K) - S(0)],$$

showing that the shaded region in IV contains P_{Eu}.

Remarks:

(i) It turns out that II and IV can be found on page 156 of Capiński and Zastawniak (2003) *Mathematics for Finance: An Introduction to Financial Engineering*, Springer Undergraduate Mathematics Series.

(ii) The last inequality in (9.9) can be derived as follows. By put-call parity,

$$C_{Eu} = P_{Eu} + F_{0,T}^{P}(S) - e^{-rT}K$$
$$\geq F_{0,T}^{P}(S) - e^{-rT}K \qquad \text{because } P_{Eu} \geq 0.$$

We also have

$$C_{Eu} \geq 0.$$

Thus,

$$C_{Eu} \geq \text{Max}[0,\ F_{0,T}^{P}(S) - e^{-rT}K].$$

(iii) An alternative derivation of the inequality above is to use *Jensen's Inequality* (see, in particular, page 883).

$$C_{Eu} = \text{E}^*\left[e^{-rT}\text{Max}(0, S(T) - K)\right]$$
$$\geq e^{-rT}\text{Max}(0, \text{E}^*\left[S(T) - K\right]) \quad \text{because of Jensen's Inequality}$$
$$= \text{Max}(0, \text{E}^*\left[e^{-rT}S(T)\right] - e^{-rT}K)$$
$$= \text{Max}(0, F_{0,T}^{P}(S) - e^{-rT}K).$$

Here, E* signifies risk-neutral expectation.

(iv) That $C_{Eu} = C_{Am}$ for nondividend-paying stocks can be shown by Jensen's Inequality.