

Solution to (31)

Answer: (B)

Assume that the bull spread is constructed by buying a 50-strike call and selling a 60-strike call. (You may also assume that the spread is constructed by buying a 50-strike put and selling a 60-strike put.)

Delta for the bull spread is equal to

$$(\text{delta for the 50-strike call}) - (\text{delta for the 60-strike call}).$$

(You get the same delta value, if put options are used instead of call options.)

$$\text{Call option delta} = N(d_1), \text{ where } d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

50-strike call:

$$d_1 = \frac{\ln(50/50) + (0.05 + \frac{1}{2} \times 0.2^2)(3/12)}{0.2\sqrt{3/12}} = 0.175, \quad N(0.175) = 0.56946$$

60-strike call:

$$d_1 = \frac{\ln(50/60) + (0.05 + \frac{1}{2} \times 0.2^2)(3/12)}{0.2\sqrt{3/12}} = -1.64822, \quad N(-1.64822) = 0.04965$$

$$\text{Delta of the bull spread} = 0.56946 - 0.04965 = 0.51981.$$

After one month, 50-strike call:

$$d_1 = \frac{\ln(50/50) + (0.05 + \frac{1}{2} \times 0.2^2)(2/12)}{0.2\sqrt{2/12}} = 0.1428869 \quad N(0.14289) = 0.55681$$

60-strike call:

$$d_1 = \frac{\ln(50/60) + (0.05 + \frac{1}{2} \times 0.2^2)(2/12)}{0.2\sqrt{2/12}} = -2.090087 \quad N(-2.0901) = 0.01830$$

$$\text{Delta of the bull spread after one month} = 0.55681 - 0.01830 = 0.53851.$$

$$\text{The change in delta} = 0.53851 - 0.51981 = 0.0187 \approx 0.02.$$