Solution to (38) Answer: (B)

The term "delta-gamma approximations for bonds" can be found on page 784 of McDonald (2006).

By Taylor series,

$$P(t, T, r_0 + \varepsilon) \approx P(t, T, r_0) + \frac{1}{1!}P_r(t, T, r_0)\varepsilon + \frac{1}{2!}P_{rr}(t, T, r_0)\varepsilon^2 + \dots ,$$

where

$$P_r(t, T, r) = -A(t, T)B(t, T)e^{-B(t, T)r} = -B(t, T)P(t, T, r)$$

and

$$P_{rr}(t, T, r) = -B(t, T)P_r(t, T, r) = [B(t, T)]^2 P(t, T, r).$$

Thus,

$$\frac{P(t,T,r_0+\varepsilon)}{P(t,T,r_0)} \approx 1 - B(t, T)\varepsilon + ½[B(t, T)\varepsilon]^2 + \dots$$

and

$$\frac{P_{Est}(0,3,0.03)}{P(0,3,0.05)} = 1 - (2 \times -0.02) + ½(2 \times -0.02)^2$$

$$= 1.0408$$