

27.

Can be calculated using risk-neutral probabilities of stock price changes and option payoffs.

Let p_1^* , p_2^* and p_3^* be the probabilities of outcome 1, 2, and 3, respectively. Y only has value under outcome 3, so

$$\underbrace{300 p_3^*}_{\text{expected value of future price of } Y \text{ at } t=1} = \underbrace{100 e^{.10}}_{\text{current price of } Y, \text{ accumulated}} \longrightarrow p_3^* = e^{.1/3} = .36839$$

X has value under outcomes 1 and 2, so

$$200 p_1^* + 50 p_2^* = 100 e^{.1}$$

$$\text{where } p_1^* + p_2^* + p_3^* = 1, \text{ so}$$

$$200 p_1^* + 50(1 - p_1^* - p_3^*) = 100 e^{.1}$$

($p_3^* = .36839$)

$$200 p_1^* + 50(.63161 - p_1^*) = 100 e^{.1}$$

$$150 p_1^* = 100 e^{.1} - 31.5805$$

$$p_1^* = .526244$$

$$p_2^* = 1 - .526244 - .36839 = .105366$$

Next, determine the payoffs under each outcome for value of $P_y - C_x$:

Outcome 1:

$$\begin{aligned} P_y - C_x &= \max(0, 95 - 0) - \max(0, 200 - 95) \\ &= 95 - 105 \\ &= -10 \end{aligned}$$

Outcome 2:

$$\begin{aligned} P_y - C_x &= \max(0, 95 - 0) - \max(0, 50 - 95) \\ &= 95 - 0 \\ &= 95 \end{aligned}$$

Outcome 3:

$$\begin{aligned} P_y - C_x &= \max(0, 95 - 300) - \max(0, 0 - 95) \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

Now use the risk-neutral probabilities to calculate the expected value of $P_y - C_x$ (discounted to today):

$$P_y - C_x = e^{-0.1} (-10 p_1^* + 95 p_2^*) = 4.2955$$