#28. Solution

Step 1) The payoff 100 occurs when $[S(1)]^2 > 100 \Rightarrow S(1) > 10$,

which is a cash-or-nothing option with strike price 10.

At time 0, the price of the option $= 100\, e^{-r \cdot T} \cdot N(d_2)$ ①

Step 2. To find # of shares in the delta-hedge program,

we differentiate ① with respect to S:

$$\frac{\partial}{\partial S} 100\, e^{-r \cdot t} \cdot N(d_2)$$

$$= 100\, e^{-r \cdot t} \cdot N'(d_2) \cdot \frac{\partial d_2}{\partial S} \quad ②$$

Recall $d_2 = \dfrac{\ln\left(\frac{S_0}{K}\right) + (r - \delta - 0.5\sigma^2) \cdot t}{\sigma \cdot \sqrt{t}}$

$$= \frac{0 + (0.02 - 0 + 0.5 \times 0.2^2) \times 1}{0.2 \times \sqrt{1}}$$

$$= 0$$

Given
$$\boxed{\begin{array}{c} T = 1 \\ r = 0.02 \\ \delta = 0 \\ \sigma = 0.2 \\ S_0 = 10 \\ K = K_1 = 10 \end{array}}$$

#28 Solution continued

$$\because \frac{\partial d_2}{\partial S} = \frac{1}{S_0 \cdot \sigma \cdot \sqrt{t}} = \frac{1}{10 \times 0.2 \times \sqrt{1}} = 0.5 \quad ③$$

$$\therefore N'(d_2) = N'(0) = \frac{e^{-\frac{0^2}{2}}}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}} \quad ④$$

plug ③ & ④ into ②:

$$\text{\# of shares of stock} = 100 \times e^{-0.02 \times 1} \times \frac{1}{\sqrt{2\pi}} \times 0.5 = 19.55$$

$\therefore$ A is the answer.