$$\boxed{\begin{array}{l} \text{i) } P(t,T,r) = A(t,T)\exp[-B(t,T)r] \\ \text{ii) } B(0,3) = 2 \end{array}}$$

#38 Peter Kissinger

Delta-gamma approximation for bonds is

$$P(t,T,r_0+\epsilon) \approx P(t,T,r_0) + P_r(t,T,r_0)\,\epsilon + \tfrac{1}{2}P_{rr}(t,T,r_0)\,\epsilon^2$$

If we let ϵ = the change in the interest rate, then since we are asked to estimate $P(\underset{t_0}{0},\underset{t_1}{3},\underset{r}{.03})$ from $P(0,3,.05)$

we have $\epsilon = .03 - .05 = -.02$

Derive P_r by taking derivative of $P(t,T,r)$ with respect to r.

Since $P(t,T,r) = A(t,T)\exp[-B(t,T)r]$

$$\frac{d\,P(t,T,r)}{dr} = -B(t,T)A(t,T)\exp[-B(t,T)r]$$

$$= -B(t,T)\,P(t,T,r)$$

$$= -2\,P(t,T,r)$$

Derive P_{rr} by taking second derivative of $P(t,T,r)$ with respect to r

Given $\frac{d\,P(t,T,r)}{dr} = -B(t,T)\,A(t,T)\exp[-B(t,T)r]$

$$\frac{d^2 P(t,T,r)}{dr^2} = B(t,T)^2 A(t,T)\exp[-B(t,T)r]$$

$$= B(t,T)^2\, P(t,T,r)$$

$$\frac{\cancel{P_{EST}(0,3,.03)}}{P(0,3,.05)} = \cancel{P(0,3,.05)} +$$

$$= 2^2\, P(t,T,r)$$

$$\frac{P_{EST}(0,3,.03)}{P(0,3,.05)} = \frac{P(0,3,.05) + (-2)P(0,3,.05)(-.02) + \frac{1}{2}(4)(-.02^2)P(0,3,.05)}{P(0,3,.05)}$$

$$= \frac{P(0,3,.05)}{P(0,3,.05)} + \frac{(-2)(-.02)P(0,3,.05)}{P(0,3,.05)} + \frac{\frac{1}{2}(4)(-.02^2)P(0,3,.05)}{P(0,3,.05)}$$

$$= 1 + (-2)(-.02) + (\tfrac{1}{2})(4)(-.02^2)$$

$$= 1 + .04 + .0008 \qquad = 1.0408 \Rightarrow \boxed{B}$$