#41 Solution

Given:
$S_0 = 45$
$\sigma = .25$
$\delta = .03$
$r = .07$
$t = 1 \text{ yr}$

Since time $\underline{\text{Contingent}}$ the present value of the option $= \min(42, S_0)$

From the graph, one can recognize that this option is a $\underline{\text{put}}$.
A put's payoff is $\max(0, K - S_t) = \max(0, 42 - S_t)$

So the contingent present value $= Ke^{-rt} - \max(0, K - S_t)$

$$V_0 = 42e^{-.07(1)} - \underbrace{\max(0, 42 - S_t)}_{\text{Premium of a put}}$$

↳ Long Bond and Short Put ($K = 42$)

$\Omega = \text{Elasticity} = \dfrac{\Delta S_0}{V_0}$

$\Delta_{put} = e^{-\delta t} N(-d_1)$

$$d_1 = \frac{\ln(S_0/K) + [r - \delta + \sigma^2(.5)]t}{\sqrt{t}\,\sigma} = \frac{\ln(45/42) + [.07 - .03 + .25^2(.5)](1)}{\sqrt{1}\,(.25)}$$

$= .56097$

$d_2 = d_1 - \sigma\sqrt{t} = .56097 - .25\sqrt{1} = .31097$

$N(-d_1) = .2877$ $N(-d_2) = .3783$ Using Normal Table

$N(-d_1) = .28741$ $N(-d_2) = .37741$ Using Normal Calculator

$\Delta_{put} = e^{-\delta t} N(-d_1) = e^{-.03(1)}(.28741) = .278916$

$$\text{Put Premium} = Ke^{-rt}N(-d_2) - S_0e^{-\delta t}N(-d_1)$$

$$= 42e^{-.07(1)}(.37791) - 45e^{-.03(1)}(.28741) = 2.2479$$

$$V_0 = Ke^{-rt} - \text{Put Premium} = 42e^{-.07(1)} - 2.2479$$

$$= 36.9126$$

$\Delta_{Bond} = 0$ so Δ_p is used as total Δ

$$\Omega = \frac{\Delta_p S_0}{V_0}$$

$\Delta_p = .278916 \qquad S_0 = 45 \qquad V_0 = 36.9126$

$$\Omega = \frac{(.278916)(45)}{36.9126} = .340025 \rightarrow \text{C}$$

NOTE:

Could have also done with Long Pre-Paid Forward + Short Call

$$V_0 = F^P_{0,1}(S) - C(S_0 = 45, K = 42, t = 1, \sigma = .25, r = .07, \delta = .03)$$

$$= 45e^{-.03} - 6.757 = 36.913$$

$$\Delta_V = \Delta_{FP} - \Delta_C = e^{-.03} - e^{-.03}N(d_1) = e^{-.03} - e^{-.03}(.71259)$$

$$\Delta_V = .278916$$

$$\Omega_V = \frac{S_0\Delta_V}{V_0} = \cancel{45(36.913)}$$

$$= \frac{45(.278916)}{36.913} = .3400 \quad \text{C}$$