

Exam MFE: Sample Question # 75

Recall From McDonald (2006) page 632:

if $x^* = \bar{x} + \beta(y - \bar{y})$, then $\text{Var}(x^*) = \text{Var}(\bar{x}) + \beta^2 \text{Var}(\bar{y}) - 2\beta \text{Cov}(\bar{x}, \bar{y})$

$$\text{AND } \beta = \frac{\text{Cov}(\bar{x}, \bar{y})}{\text{Var}(\bar{y})}$$

The minimum $\text{Var}(x^*)$ is:

$$\text{Var}(x^*) = \text{Var}(\bar{x}) + \left[\frac{\text{Cov}(\bar{x}, \bar{y})}{\text{Var}(\bar{y})} \right]^2 \cdot \text{Var}(\bar{y}) - 2 \left[\frac{\text{Cov}(\bar{x}, \bar{y})}{\text{Var}(\bar{y})} \right] \cdot \text{Cov}(\bar{x}, \bar{y})$$

$$= \text{Var}(\bar{x}) + \frac{\text{Cov}(\bar{x}, \bar{y})^2}{\text{Var}(\bar{y})} - 2 \cdot \frac{\text{Cov}(\bar{x}, \bar{y})^2}{\text{Var}(\bar{y})}$$

$$= \text{Var}(\bar{x}) - \frac{\text{Cov}(\bar{x}, \bar{y})^2}{\text{Var}(\bar{y})}$$

$$= \text{Var}(\bar{x}) \left[1 - \frac{\text{Cov}(\bar{x}, \bar{y})^2}{\text{Var}(\bar{x}) \text{Var}(\bar{y})} \right]$$

$$= \text{Var}(\bar{x}) \left[1 - \left[\frac{\text{Cov}(\bar{x}, \bar{y})}{\text{Std}(\bar{x}) \text{Std}(\bar{y})} \right]^2 \right]$$

$$= \text{Var}(\bar{x}) \left[1 - \text{Corr}(\bar{x}, \bar{y})^2 \right]$$

$$= 5^2 [1 - .8^2]$$

$$= 25 [.36]$$

$$= 9$$

Recall:

$$\text{Corr}(x, y) = \frac{\text{Cov}(x, y)}{\text{Std}(x) \text{Std}(y)}$$