

Question #3

Answer: D

$$\begin{aligned} E[Z] &= \int_0^{\infty} b_t v^t {}_t p_x \mu_{x+t} dt = \int_0^{\infty} e^{0.06t} e^{-0.08t} e^{-0.05t} \frac{1}{20} dt \\ &= \frac{1}{20} \left(\frac{100}{7} \right) \left[-e^{-0.07t} \right]_0^{\infty} = \frac{5}{7} \end{aligned}$$

$$\begin{aligned} E[Z^2] &= \int_0^{\infty} (b_t v^t)^2 {}_t p_x \mu_{x+t} dt = \int_0^{\infty} e^{0.12t} e^{-0.16t} e^{-0.05t} \frac{1}{20} dt = \frac{1}{20} \int_0^{\infty} e^{-0.09t} dt \\ &= \frac{1}{20} \left(\frac{100}{9} \right) \left[e^{-0.09t} \right]_0^{\infty} = \frac{5}{9} \end{aligned}$$

$$\text{Var}[Z] = \frac{5}{9} - \left(\frac{5}{7} \right)^2 = 0.04535$$