**Question #16**
**Answer: A**

$$1000P_{40}=\frac{A_{40}}{\ddot{a}_{40}}=\frac{161.32}{14.8166}=10.89$$

$$1000\,_{20}V_{40}=1000\left(1-\frac{\ddot{a}_{60}}{\ddot{a}_{40}}\right)=1000\left(1-\frac{11.1454}{14.8166}\right)=247.78$$

$$_{21}V=\frac{\left(_{20}V+5000P_{40}\right)(1+i)-5000q_{60}}{P_{60}}$$

$$=\frac{\left(247.78+(5)(10.89)\right)\times 1.06-5000\left(0.01376\right)}{1-0.01376}=255$$

[Note: For this insurance, $_{20}V=1000\,_{20}V_{40}$ because retrospectively, this is identical to whole life]

Though it would have taken <u>much</u> longer, you can do this as a prospective reserve. The prospective solution is included for educational purposes, not to suggest it would be suitable under exam time constraints.

$1000P_{40}=10.89$ as above

$$1000A_{40}+4000\,_{20}E_{40}A^{1}_{60:\overline{5|}}=1000P_{40}+5000P_{40}\times\,_{20}E_{40}\,\ddot{a}_{60:\overline{5|}}+\pi\,_{20}E_{40}\times\,_{5}E_{60}\,\ddot{a}_{65}$$

where $A^{1}_{60:\overline{5}|} = A_{60} - {}_{5}E_{60}\ A_{65} = 0.06674$

$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40}\ \ddot{a}_{60} = 11.7612$

$\ddot{a}_{60:\overline{5}|} = \ddot{a}_{60} - {}_{5}E_{60}\ \ddot{a}_{65} = 4.3407$

$$1000(0.16132)+(4000)(0.27414)(0.06674)=$$

$$=(10.89)(11.7612)+(5)(10.89)(0.27414)(4.3407)+\pi(0.27414)(0.68756)(9.8969)$$

$$\pi = \frac{161.32+73.18-128.08-64.79}{1.86544}$$
$$= 22.32$$

Having struggled to solve for $\pi$, you could calculate ${}_{20}V$ prospectively then (as above)
calculate ${}_{21}V$ recursively.

$$\begin{aligned}{}_{20}V &= 4000A^{1}_{60:\overline{5}|} + 1000A_{60} - 5000P_{40}\ \ddot{a}_{60:\overline{5}|} - \pi\ {}_{5}E_{60}\ \ddot{a}_{65} \\ &= (4000)(0.06674)+369.13-(5000)(0.01089)(4.3407)-(22.32)(0.68756)(9.8969) \\ &= 247.86 \text{ (minor rounding difference from } 1000\ {}_{20}V_{40}\text{)}\end{aligned}$$

Or we can continue to ${}_{21}V$ prospectively

$${}_{21}V = 5000A^{1}_{61:\overline{4}|} + 1000\ {}_{4}E_{61}\ A_{65} - 5000P_{40}\ \ddot{a}_{61:\overline{4}|} - \pi\ {}_{4}E_{61}\ \ddot{a}_{65}$$

where ${}_{4}E_{61} = \dfrac{l_{65}}{l_{61}} v^4 = \left(\dfrac{7,533,964}{8,075,403}\right)(0.79209) = 0.73898$

$$\begin{aligned}A^{1}_{61:\overline{4}|} &= A_{61} - {}_{4}E_{61}\ A_{65} = 0.38279 - 0.73898 \times 0.43980 \\ &= 0.05779\end{aligned}$$

$$\begin{aligned}\ddot{a}_{61:\overline{4}|} &= \ddot{a}_{61} - {}_{4}E_{61}\ \ddot{a}_{65} = 10.9041 - 0.73898 \times 9.8969 \\ &= 3.5905\end{aligned}$$

$$\begin{aligned}{}_{21}V &= (5000)(0.05779)+(1000)(0.73898)(0.43980) \\ &\quad -(5)(10.89)(3.5905)-22.32(0.73898)(9.8969) \\ &= 255\end{aligned}$$

Finally. A moral victory. Under exam conditions since prospective benefit reserves must equal retrospective benefit reserves, calculate whichever is simpler.