Question #93
Answer: A

Let π be the benefit premium

Let ${}_kV$ denote the benefit reserve at the end of year *k*.

For any $n, \left({}_nV + \pi\right)\left(1+i\right) = \left(q_{25+n} \times {}_{n+1}V + p_{25+n} \times {}_{n+1}V\right)$

$$= {}_{n+1}V$$

Thus ${}_1V = \left({}_0V + \pi\right)\left(1+i\right)$

$${}_2V = \left({}_1V + \pi\right)\left(1+i\right) = \left(\pi\left(1+i\right) + \pi\right)\left(1+i\right) = \pi\ddot{s}_{\overline{2|}}$$

$${}_3V = \left({}_2V + \pi\right)\left(1+i\right) = \left(\pi\ddot{s}_{\overline{2|}} + \pi\right)\left(1+i\right) = \pi\ddot{s}_{\overline{3|}}$$

By induction (proof omitted)

$${}_nV = \pi\ddot{s}_{\overline{n|}}$$

For $n = 35,\ {}_nV = \ddot{a}_{60}$ (expected present value of future benefits; there are no future premiums)

$$\ddot{a}_{60} = \pi\ \ddot{s}_{\overline{35|}}$$

$$\pi = \frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35|}}} \quad \text{For} \quad n = 20, \quad {}_{20}V = \pi\ \ddot{s}_{\overline{20|}} = \left(\frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35|}}}\right)\ddot{s}_{\overline{20|}}$$

Alternatively, as above

$\left({}_nV+\pi\right)\left(1+i\right)={}_{n+1}V$

Write those equations, for $n=0$ to $n=34$

$$0:\left({}_0V+\pi\right)\left(1+i\right)={}_1V$$
$$1:\left({}_1V+\pi\right)\left(1+i\right)={}_2V$$
$$2:\left({}_2V+\pi\right)\left(1+i\right)={}_3V$$
$$\vdots$$
$$34:\left({}_{34}V+\pi\right)\left(1+i\right)={}_{35}V$$

Multiply equation *k* by $\left(1+i\right)^{34-k}$ and sum the results:

$$\left({}_0V+\pi\right)\left(1+i\right)^{35}+\left({}_1V+\pi\right)\left(1+i\right)^{34}+\left({}_2V+\pi\right)\left(1+i\right)^{33}+\cdots+\left({}_{34}V+\pi\right)\left(1+i\right)=$$
$${}_1V\left(1+i\right)^{34}+{}_2V\left(1+i\right)^{33}+{}_3V\left(1+i\right)^{32}+\cdots+{}_{34}V\left(1+i\right)+{}_{35}V$$

For $k=1,2,\cdots,34,$ the ${}_kV\left(1+i\right)^{35-k}$ terms in both sides cancel, leaving

$${}_0V\left(1+i\right)^{35}+\pi\left[\left(1+i\right)^{35}+\left(1+i\right)^{34}+\cdots+\left(1+i\right)\right]={}_{35}V$$

Since ${}_0V=0$

$$\pi\,\ddot{s}_{\overline{35|}}={}_{35}V$$
$$=\ddot{a}_{60}$$

(see above for remainder of solution)