Question #122A
Answer: C

Because your original survival function for (*x*) was correct, you must have

$$\mu_{x+t} = 0.06 = \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{02}_{x+t:y+t} + 0.02$$

$$\mu^{02}_{x+t:y+t} = 0.04$$

Similarly, for (*y*)

$$\mu_{y+t} = 0.06 = \mu^{01}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{01}_{x+t:y+t} + 0.02$$

$$\mu^{01}_{x+t:y+t} = 0.04$$

The first-to-die insurance pays as soon as State 0 is left, regardless of which state is next. The force of transition from State 0 is
$\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03} = 0.04 + 0.04 + 0.02 = 0.10$.

With a constant force of transition, the expected present value is
$$\int_0^{\infty} e^{-\delta t} {}_t p_{xy}^{00} (\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03}) dt = \int_0^{\infty} e^{-0.05t} e^{-0.10t} (0.10) dt = \frac{0.10}{0.15}$$

Question #122B
Answer: E

Because (*x*) is to have a constant force of 0.06 regardless of (*y*)'s status (and vice-versa) it must be that $\mu_{x+t:y+t}^{13} = \mu_{x+t:y+t}^{23} = 0.06$.

There are three mutually exclusive ways in which both will be dead by the end of year 3:

1: Transition from State 0 directly to State 3 within 3 years. The probability of this is
$$\int_0^3 {}_t p_{xy}^{00} \mu_{x+t:y+t}^{03} dt = \int_0^3 e^{-0.10t} 0.02 dt = -\frac{0.02}{0.10} e^{-0.10t} \Big|_0^3 = 0.2(1 - e^{-0.3}) = 0.0518$$
2: Transition from State 0 to State 1 and then to State 3 all within 3 years. The probability of this is
$$\int_0^3 {}_t p_{xy}^{00} \mu_{x+t:y+t}^{01} {}_{3-t} p_{x+t:y+t}^{13} dt = \int_0^3 e^{-0.10t} 0.04(1 - e^{-0.06(3-t)}) dt$$
$$= \int_0^3 0.04 \left[e^{-0.10t} - e^{-0.18} e^{-0.04t} \right] = -\frac{0.04}{0.10} e^{-0.10t} + \frac{0.04 e^{-0.18}}{0.04} e^{-0.04t} \Big|_0^3$$
$$= 0.4(1 - e^{-0.3}) - e^{-0.18}(1 - e^{-0.12}) = 0.00922$$

3: Transition from State 0 to State 2 and then to State 3 all within 3 years. By symmetry, this probability is 0.00922.

The answer is then 0.0518 + 2(0.00922) = 0.0702.

Question #122C
Answer: D

Because the original survival function continues to hold for the individual lives, with a constant force of mortality of 0.06 and a constant force of interest of 0.05, the expected present values of the individual insurances are

$$\bar{A}_x = \bar{A}_y = \frac{0.06}{0.06+0.05} = 0.54545,$$

Then,

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Alternatively, the answer can be obtained be using the three mutually exclusive outcomes used in the solution to Question 122B.

1: $\int_0^\infty e^{-0.05t}\, {}_tp_{xy}^{00} \mu_{x+t:y+t}^{03} dt = \int_0^\infty e^{-0.05t} e^{-0.10t}\, 0.02 dt = \frac{0.02}{0.15} = 0.13333$

2 and 3:
$$\int_0^\infty e^{-0.05t}\, {}_tp_{xy}^{00} \mu_{x+t:y+t}^{01} \int_0^\infty e^{-0.05r}\, {}_rp_{x+t:y+t}^{11} \mu_{x+t+r:y+t+r}^{13} drdt$$
$$= \int_0^\infty e^{-0.05t} e^{-0.10t}\, 0.04 \int_0^\infty e^{-0.05r} e^{-0.06r}\, 0.06 drdt = \frac{0.04}{0.15}\frac{0.06}{0.11} = 0.14545$$

The solution is 0.13333 + 2(0.14545) = 0.42423.

The fact that the double integral factors into two components is due to the memoryless property of the exponential transition distributions.