Question #220
Answer: C

$$\mu_x^{NS} = \frac{500}{500(110-x)} = \frac{1}{110-x}$$

$$= \tfrac{1}{2}\,\mu_x^S \Rightarrow \mu_x^S = \frac{2}{110-x}$$

$$\Rightarrow l_x^S = l_0^S\left(110-x\right)^2$$ [see note below]

Thus $${}_t p_{20}^S = \frac{l_{20+t}^S}{l_{20}^S} = \frac{\left(90-t\right)^2}{90^2}$$

$${}_t p_{25}^{NS} = \frac{l_{25+t}^{NS}}{l_{25}^{NS}} = \frac{\left(85-t\right)}{85}$$

$$\overset{\circ}{e}_{20:25} = \int_0^{85} {}_tp_{20:25}\,dt$$

$$= \int_0^{85} {}_tp_{20}^{S}\; {}_tp_{25}^{NS}\,dt = \int_0^{85} \frac{(90-t)^2}{(90)^2}\frac{(85-t)}{85}\,dt$$

$$= \frac{1}{688,500}\int_0^{85}(90-t)^2(90-t-5)\,dt$$

$$= \frac{1}{688,500}\left[\int_0^{85}(90-t)^3\,dt - 5\int_0^{85}(90-t)^2\,dt\right]$$

$$= \frac{1}{688,500}\left[\frac{-(90-t)^4}{4}+\frac{5(90-t)^3}{3}\right]_0^{85}$$

$$= \frac{1}{688,500}\left[-156.25+208.33+16,402,500-1,215,000\right]$$

= 22.1

[There are other ways to evaluate the integral, leading to the same result].

The $S_0(x)$ form is derived as $S_0(x) = e^{-\int_0^x\left(\frac{2}{110-t}\right)dt} = e^{2\ln(110-t)\big|_0^x} = \left(\frac{110-x}{110}\right)^2$

The l_x form is equivalent.