Question #306
Answer: E

$${}_tV' = \frac{d}{dt}\,{}_tV = G + \delta({}_tV) - \mu_t(b_t - {}_tV)$$

At t = 4.5,

$${}_{4.5}V' = 25 + 0.05({}_{4.5}V) - 0.02(4.5)(100 - {}_{4.5}V) = 16 + 0.14({}_{4.5}V)$$

Euler's formula in this case is ${}_{5.0}V = {}_{4.5}V + (5.0 - 4.5)\,{}_{4.5}V'$.

Because the endowment benefit is 100, ${}_{5.0}V = 100$ and thus,

$$100 = {}_{4.5}V + 0.5({}_{4.5}V') = {}_{4.5}V + 0.5[16 + 0.14({}_{4.5}V)] = 8 + 1.07({}_{4.5}V)$$

$${}_{4.5}V = 85.981.$$

Similarly,

${}_{4.5}V = {}_{4}V + (4.5 - 4.0)\,{}_{4}V'$ and

$${}_{4.0}V' = 25 + 0.05({}_{4.0}V) - 0.02(4.0)(100 - {}_{4.0}V) = 17 + 0.13({}_{4.0}V)$$

$$85.981 = {}_{4.0}V + 0.5({}_{4.0}V'') = {}_{4.0}V + 0.5[17 + 0.13({}_{4.0}V)] = 8.5 + 1.065({}_{4.0}V)$$

$${}_{4.0}V = 72.752.$$

Note that if smaller step sizes were used (which would be inappropriate for an exam question, where the step size must be specified), the estimate of the time 4 reserve converges to its true value of 71.96.