

Problem MLC 002

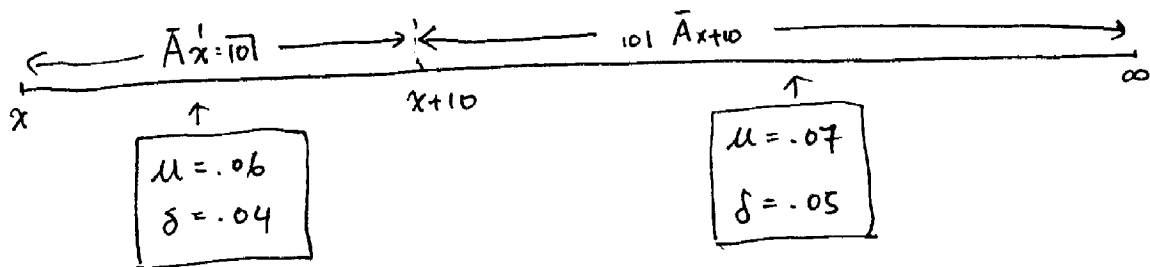
Asked to find: Single Benefit Premium

Need to Calculate: the Actuarial Present Value of the Benefit

Given: whole Life

\$1,000

moment of death



* different forces of mortality

* different interest rate

→ divide the timeline into two parts

$$\begin{aligned}
 1000 \cdot \bar{A}_x &= 1000 \cdot (\bar{A}'_{x:\overline{10}|} + {}_{10|}\bar{A}_{x+10}) \\
 &= 1000 (\int_0^{10} v^t \cdot {}_tP_x \cdot \mu_{x+t} dt + {}_{10}P_x \cdot v^{10} \cdot \int_0^{\infty} v^s \cdot {}_sP_{x+10} \cdot \mu_{x+10+s} ds) \\
 &= 1000 \cdot (\int_0^{10} e^{-.04t} \cdot e^{-.06t} \cdot (.06) dt + e^{-.06 \times 10} \cdot e^{-.04 \times 10} \cdot \int_0^{\infty} e^{-.05s} \cdot e^{-.07s} (.07) ds) \\
 &= 1000 \cdot (0.6 \cdot \int_0^{10} e^{-.1t} dt + e^{-1} (0.07) \cdot \int_0^{\infty} e^{-.12s} ds) \\
 &= 1000 (\frac{0.6}{0.1} (1 - e^{-1}) + e^{-1} \frac{0.07}{0.12} (1 - e^{-1.2})) \\
 &= 593.87 \quad \text{E}
 \end{aligned}$$

Alternative Method

* Constant force of mortality & interest rate

$$\bar{A}_x = \frac{\mu}{\mu + \delta}$$

$$\bar{A}_{x+10} = \bar{A}_x = \frac{\mu}{\mu + \delta}$$

$$\begin{aligned}\bar{A}'_{x:\overline{10}|} &= \bar{A}_x - {}_{10}E_x \bar{A}_{x+10} \\ &= \frac{\mu}{\mu + \delta} (1 - {}_{10}P_x \cdot V^{10}) \\ &= \frac{\mu}{\mu + \delta} (1 - e^{-10(\mu + \delta)})\end{aligned}$$

$${}_{10|}\bar{A}_x = {}_{10}E_x \cdot \bar{A}_{x+10}$$

$$= e^{-10(\mu + \delta)} \frac{\mu^*}{\mu^* + \delta^*} \quad * \text{ new mortality and interest rate}$$

$$1000 \bar{A}_x = 1000 (\bar{A}'_{x:\overline{10}|} + {}_{10}E_x \cdot \bar{A}_{x+10})$$

$$= 1000 \left(\frac{\mu}{\mu + \delta} (1 - e^{-10(\mu + \delta)}) + e^{-10(\mu + \delta)} \cdot \frac{\mu^*}{\mu^* + \delta^*} \right)$$

$$= 1000 \left(\frac{0.06}{0.06 + 0.04} (1 - e^{-1}) + e^{-1} \cdot \frac{0.07}{0.07 + 0.05} \right)$$

$$= 593.87$$

(E)