

Problem MLC 003

Given: Benefit = b_t

$$\mu_{x+t} = .05$$

$$\delta = .08$$

whole life

moment of death

$$Z = b_t \cdot V^t$$

Asked: $\text{Var}(Z)$

$$\text{Var}(Z) = E(Z^2) - (E(Z))^2$$

$$\begin{aligned} E(Z) &= \int_0^{\infty} b_t \cdot V^t \cdot {}_tP_x \cdot \mu_{x+t} dt \\ &= \int_0^{\infty} e^{-.06t} \cdot e^{-.08t} \cdot e^{-.05t} (0.05) dt \\ &= \frac{1}{20} \cdot \frac{100}{7} \cdot (-e^{-.09t}) \Big|_0^{\infty} \\ &= \frac{5}{7} \end{aligned}$$

$$\begin{aligned} E(Z^2) &= \int_0^{\infty} (b_t \cdot V^t)^2 \cdot {}_tP_x \cdot \mu_{x+t} dt \\ &= \int_0^{\infty} (e^{-.12t} \cdot e^{-.08t})^2 \cdot e^{-.05t} \cdot (0.05) dt \\ &= \frac{1}{20} \left(\frac{-e^{-.09t}}{.09} \right) \Big|_0^{\infty} \\ &= \frac{5}{9} \end{aligned}$$

$$\Rightarrow \text{Var}(Z) = \frac{5}{9} - \left(\frac{5}{7}\right)^2 = .04535$$