MLC Solution #45

$b_0 \ddot{a}_{50} = 50000$ $\quad$ $\mathring{e}_y = 15$

$\mathring{e}_{50} = 25$ $\quad$ $i = 0.06$

$l_x = 100(\omega - x)$ for $0 \le x \le \omega$

$\Rightarrow$ De moivre: ${}_tp_x = \dfrac{\omega - x - t}{\omega - x}$

Goal

$b\ddot{a}_y = 50000$

$b = ?$

Average annuitant

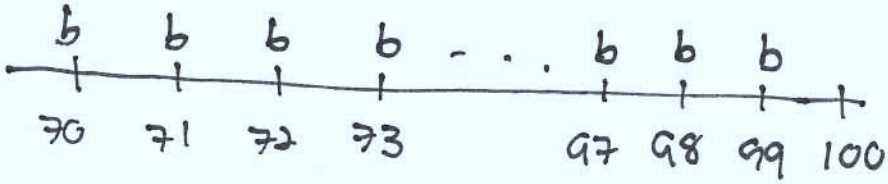
$$\mathring{e}_{50} = \int_0^{\omega-50} 1 - \frac{t}{\omega-50}\,dt = t - \frac{t^2}{2(\omega-50)}\Big|_0^{\omega-50} = \frac{\omega-50}{2} = 25$$

$\Rightarrow \omega = 100$

$$\mathring{e}_y = \int_0^{100-y} 1 - \frac{t}{100-y} = \frac{100-y}{2} = 15 \Rightarrow y = 70$$

$$\ddot{a}_{70} = \frac{1 - A_{70}}{d} = \sum_{k=0}^{29} v^k \, {}_kp_x$$

Special annuitant



$$A_x = \sum_{k=0}^{n-1} v^{k+1} \, {}_{k|}q_x$$

Demoivre $\Rightarrow$ $${}_{k|}q_x = \frac{100-x-k}{100-x} \cdot \frac{1}{100-x-k} = \frac{1}{100-x}$$

$$A_{70} = \sum_{k=0}^{100-70-1} v^{k+1} \, {}_{k|}q_{70} = \frac{1}{30} \sum_{k=0}^{29} v^{k+1}$$

$$= \frac{a_{\overline{30}|}}{30} = \frac{13.76483}{30} = 0.45883$$

$$\sum_{k=0}^{29} v^{k+1} = v + v^2 + \ldots + v^{29} + v^{30} = a_{\overline{30}|}$$

$$\ddot{a}_{70} = \frac{1 - 0.45883}{\frac{0.06}{1.06}} = 9.56135$$

$$b\,\ddot{a}_{70} = b(9.56135) = 500000$$

$$\Rightarrow b = 52293.87 \approx \boxed{52000} \quad \boxed{E}$$