MLC # 94

Given:

Future lifetimes independent

Mortality ILT

UDD Approx.

Want to find $\mu_{\overline{50:50}}(10.5)$

Recall:

$$\mu_x = \frac{f_x(t)}{S_x(t)} = \frac{-\frac{d}{dt} S_x(t)}{S_x(t)}$$

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}$$

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_x \cdot {}_tp_y$$

$$\mu_{\overline{xy}}(t) = \frac{-\frac{d}{dt}\,{}_tp_{\overline{xy}}}{{}_tp_{\overline{xy}}}$$

$$
\begin{aligned}
-\frac{d}{dt}\,{}_tp_{\overline{xy}} &= -\frac{d}{dt}\left({}_tp_x + {}_tp_y - {}_tp_x \cdot {}_tp_y\right) \\
&= -\left(-{}_tp_x\,\mu_{x+t} - {}_tp_y\,\mu_{y+t} - \left(-{}_tp_x \cdot {}_tp_y \cdot \mu_{x+t} - {}_tp_x \cdot {}_tp_y \cdot \mu_{y+t}\right)\right) \\
&= {}_tp_x\,\mu_{x+t} + {}_tp_y\,\mu_{y+t} - \left({}_tp_x\,{}_tp_y\,\mu_{x+t} + {}_tp_x\,{}_tp_y\,\mu_{y+t}\right) \\
&= {}_tp_x\,\mu_{x+t} - {}_tp_x\,{}_tp_y\,\mu_{x+t} + {}_tp_y\,\mu_{y+t} - {}_tp_x\,{}_tp_y\,\mu_{y+t} \\
&= {}_tp_x\,\mu_{x+t}(1 - {}_tp_y) + {}_tp_y\,\mu_{y+t}(1 - {}_tp_x)
\end{aligned}
$$

$$\mu_{\overline{xy}}(t) = \frac{{}_tp_x\,\mu_{x+t}(1-{}_tp_y) + {}_tp_y\,\mu_{y+t}(1-{}_tp_x)}{{}_tp_x + {}_tp_y - {}_tp_x \cdot {}_tp_y}$$

$$= \frac{2\,{}_tp_x\,\mu_{x+t}(1-{}_tp_x)}{2\,{}_tp_x - ({}_tp_x)^2}$$

$$\mu_{\overline{50:50}}(10.5) = \frac{2\,{}_{10.5}p_{50}\,\mu_{60.5}(1-{}_{10.5}p_{50})}{2\,{}_{10.5}p_{50} - ({}_{10.5}p_{50})^2}$$

since UDD

$${}_{10.5}p_{50} = \frac{l_{60.5}}{l_{50}} = \frac{\frac{1}{2}(l_{60}+l_{61})}{l_{50}} = \frac{\frac{1}{2}(8188074+8075403)}{8950901}$$

$${}_{10.5}p_{50} = .90848$$

recall: $\mu_{x+s} = \dfrac{q_x}{1-s\,q_x}$ $\quad 0<s<1$ for UDD

$$\mu_{60.5} = \frac{q_{60}}{1-\frac{1}{2}q_{60}} = \frac{.01376}{1-\frac{1}{2}(.01376)} = .013855$$

$$\mu_{\overline{50:50}}(10.5) = \frac{2(.90848)(.013855)(1-.90848)}{2(.90848)-(.90848)^2} = .00232 \qquad \boxed{B}$$