

Solution #140

$$\text{Var}(Y) = E[Y^2] - E[Y]^2$$

$$E[Y^n] = \sum_{k=0}^{\infty} k | q_x \cdot Y^n$$

$$\Pr(k=0) = 0 | q_x = q_x = (1-0.9) = 0.1$$

$$\Pr(k=1) = 1 | q_x = p_x(1-p_{x+1}) = 0.9(1 - \frac{0.9^2}{0.9}) = 0.09$$

Since the annuitant only needs to survive 2 full years to receive the third and final payment, we need to find $\Pr(K \geq 2)$, not $\Pr(K=2)$

$$\Pr(K \geq 2) = {}_2p_x = 0.9^2 = 0.81$$

Probability	Y	Y ²
0.1	1.00	1.00 ²
0.09	1.87	1.87 ²
0.81	2.72	2.72 ²

$$E[Y] = 0.1(1) + 0.09(1.87) + 0.81(2.72) = 2.4715$$

$$E[Y^2] = 0.1(1^2) + 0.09(1.87^2) + 0.81(2.72^2) = 6.407$$

$$\text{Var}(Y) = 6.407 - 2.4715^2$$

$$= 0.30 \quad \textcircled{B}$$