

Problem MLC #144

$l_0^{(\tau)}$: number of students entering year 1

$l_1^{(\tau)}$: number of students entering year 2

$l_2^{(\tau)}$: number of students entering year 3

f : academic failure

w : withdrawal

$$p_0^{(\tau)} = 1 - 0.40 - 0.20 = 0.40$$

$$l_2^{(\tau)} = 10 \cdot l_1^{(\tau)} q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$$

$$q_2^{(w)} = q_2^{(\tau)} - q_2^{(f)} = (1.0 - 0.6) - 0.1 = 0.3$$

$$l_1^{(\tau)} q_1^{(f)} = 0.4 [l_1^{(\tau)} (1 - q_1^{(f)} - q_1^{(w)})]$$

$$q_1^{(f)} = 0.4 (1 - q_1^{(f)} - 0.3)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(\tau)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$$\begin{aligned} {}_3q_0^{(w)} &= q_0^{(w)} + p_0^{(\tau)} q_1^{(w)} + p_0^{(\tau)} p_1^{(\tau)} q_2^{(w)} \\ &= 0.2 + (0.4)(0.3) + (0.4)(0.5)(0.3) \\ &= 0.38 \end{aligned}$$

Alternative Method

	Pr (fail)	Pr (Withdrawal)	Pr (Entering Next Year)
Yr1	0.4	0.2	A
Yr2	B	0.3	C
Yr3	D	E	0.6

$$0.4 + 0.2 + A = 1 \Rightarrow A = 0.4$$

$$\left. \begin{array}{l} B + 0.3 + C = 1 \\ B = 0.4 \cdot C \end{array} \right\} \Rightarrow \begin{array}{l} B = 0.2 \\ C = 0.5 \end{array}$$

$$\left. \begin{array}{l} D = 0.1 \\ E + D + 0.6 = 1 \end{array} \right\} \Rightarrow \begin{array}{l} D = 0.1 \\ E = 0.3 \end{array}$$

$$\begin{aligned} \text{Pr (withdrawal in all 3 years)} &= \text{Pr (withdrawal in Yr1)} + \\ &\quad \text{Pr (withdrawal in Yr2 under condition "Pass Yr1")} \\ &\quad + \text{Pr (withdrawal in Yr3 under condition "Pass Yr1,2")} \end{aligned}$$

$$= 0.2 + 0.4 \times 0.3 + 0.4 \times 0.5 \times 0.3$$

$$= 0.38$$

B