Solution 150

$\mu_x = \frac{1}{100-x} \quad 0 \le x < 100$ This is De Moivre with $w = 100$

so ${}_tp_x = \frac{w-x-t}{w-x}$ and $\mathring{e}_x = \frac{w-x}{2}$

$\mathring{e}_{\overline{50:60}} = \mathring{e}_{50} + \mathring{e}_{60} - \mathring{e}_{50:60}$

$\mathring{e}_{50} = \frac{100-50}{2} = 25 \quad \mathring{e}_{60} = \frac{100-60}{2} = 20$

$\mathring{e}_{50:60} = \int_0^{40} {}_tp_{50} \cdot {}_tp_{60}\, dt = \int_0^{40} \left(\frac{50-t}{50}\right)\left(\frac{40-t}{40}\right) dt$

$= \int_0^{40} \frac{2000 - 90t + t^2}{2000}\, dt$

$= \frac{1}{2000}\left[2000t - 45t^2 + \frac{t^3}{3}\right]_0^{40} = 14.\overline{6}$

$\mathring{e}_{\overline{50:60}} = 25 + 20 - 14.\overline{6} = 30.\overline{3} \approx 30 \quad \boxed{A}$

We can check the De Moivre formulas

${}_tp_x = \exp - \int_0^t \mu_{x+s}\, ds$

${}_tp_x = \exp - \int_0^t \frac{1}{w-x-s}\, ds = \exp - \left[-\ln(w-x-s)\right]_0^t$

$= \exp\left[\ln(w-x-t) - \ln(w-x)\right] = \exp\left[\ln\left(\frac{w-x-t}{w-x}\right)\right]$

$= \frac{w-x-t}{w-x}$

$\mathring{e}_x = \int_0^\infty {}_tp_x\, dt$

$\mathring{e}_x = \int_0^{w-x} \frac{w-x-t}{w-x}\, dt = \left[\frac{(w-x)t - \frac{t^2}{2}}{w-x}\right]_0^{w-x}$

$= \frac{(w-x)^{\cancel{2}} - \frac{(w-x)^{\cancel{2}}}{2}}{\cancel{w-x}} = \frac{w-x}{2}$