MLC #176

Z = Present Value of Death Benefit $= v^t \cdot b_t$

$$Var(z) = E(Z^2) - [E(z)]^2$$

$$E(Z) = \int_0^\infty (v^t \cdot b_t) \cdot {}_tp_x \cdot \mu_{x+t}\, dt$$

$\hookrightarrow$ If $b_t = 1$, $E(Z) = \bar{A}_x$

$$= \int_0^\infty e^{-0.08t} \cdot e^{0.03t} \cdot e^{-0.02t} \cdot 0.02\, dt$$

$$= \int_0^\infty e^{-0.07t} \cdot 0.02\, dt = \frac{0.02}{-0.07} e^{-0.07t} \Big|_0^\infty = 2/7$$

$$E(Z^2) = \int_0^\infty (v^t b_t)^2 \cdot {}_tp_x \cdot \mu_{x+t}\, dt$$

$$= \int_0^\infty (e^{-0.08t} \cdot e^{0.03t})^2 \cdot e^{-0.02t} \cdot 0.02\, dt$$

$$= \int_0^\infty e^{-0.16t} \cdot e^{0.06t} \cdot e^{-0.02t} \cdot 0.02\, dt$$

$$= \int_0^\infty e^{-0.12t} \cdot 0.02\, dt = \frac{0.02}{-0.12} e^{-0.12t} \Big|_0^\infty = 2/12$$

$$Var(z) = 1/6 - (2/7)^2 = 0.08503 \text{ (C)}$$

$$E[x] = \int_{-\infty}^\infty x \cdot f(x)\, dx$$

$\uparrow$ probability density function

$v^t = e^{-\delta t}$

${}_tp_x = e^{-\mu t}$ when μ is constant.