

MLC Question 249 .

X - Uniform distribution of deaths over each year (UDD)

Y - Constant force of mortality (CF) of 0.25

$q_{\overline{xy}} = 0.125$ - probability that X dies first within the coming year

$$q_{\overline{xy}} = \int_0^1 t p_{xy} \cdot \mu_{x+t} dt$$

$$= \int_0^1 t p_y \cdot \underbrace{t p_x \cdot \mu_{x+t}}_{\text{probability density function for the mortality of } x} dt$$

because $T(x)$ and $T(y)$ are independent

$$= q_x \int_0^1 t p_y dt$$

because the mortality of X is UDD

$$= q_x \int_0^1 e^{-0.25t} dt$$

because $T(y)$ follows CF of 0.25,
therefore $t p_y = e^{-\mu_y \cdot t} = e^{-0.25t}$

$$= q_x \left[\frac{e^{-0.25t}}{-0.25} \right]_0^1$$

$$= q_x \left[\frac{1 - e^{-0.25}}{0.25} \right]$$

$$= 0.884797 q_x = 0.125$$

Therefore $q_x = 0.141$

(B)