Solution #263

$q_{30} = 0.4$

$q_{40} = 0.6$

Deaths are uniformly distributed between integer ages

$_{0.25}q^{\;2}_{30.5:40.5} = Pr($ Both (30.5) and (40.5) die within 0.25 years, with (30.5) dying second$)$

$$_{0.25}q^{\;2}_{30.5:40.5} = \int_0^{0.25} {}_tq_{40.5} \; {}_tp_{30.5} \, \mu_{30.5+t} \, dt$$

* Under UDD assumption

$${}_tq_x = t \cdot q_x$$

$${}_tp_x \, \mu_{x+t} = \frac{d}{dt} {}_tq_x = q_x$$

${}_tp_x \, \mu_{x+t}$ is the density function of T_x

${}_tq_x$ is the distribution function of T_x

$$ {}_tq_{40.5} = {}_tq_{40} / {}_{0.5}p_{40} = t \cdot q_{40} / (1 - 0.5 q_{40}) $$

$$ = \frac{0.6t}{1-0.5(0.6)} = \frac{0.6t}{0.7} $$

$$ {}_tp_{30.5}\,\mu_{30.5+t} = {}_tp_{30.5} \cdot \frac{\frac{d}{dt}\,{}_tq_{30.5}}{{}_tp_{30.5}} $$

$$ = \frac{d}{dt}\left({}_tq_{30} / {}_{0.5}p_{30}\right) $$

$$ = \frac{d}{dt}\left(t \cdot q_{30} / 1 - .5q_{30}\right) $$

$$ = q_{30} / 1 - 0.5q_{30} $$

$$ = 0.4 / (1 - \tfrac{1}{2}(0.4)) $$

$$ = 1/2 $$

$$ {}_{0.25}q^{\;2}_{30.5:40.5} = \int_0^{0.25} \frac{0.6t}{0.7}\left(\frac{1}{2}\right) dt $$

$$ = \int_0^{0.25} \left(\frac{3}{7}\right) t \, dt $$

$$ = \left(\frac{3}{7}\right) \frac{t^2}{2} \Big|_0^{0.25} $$

$$ = \frac{3}{7} \cdot \frac{1}{2} \cdot \left(\frac{1}{4}\right)^2 $$

$$ = 0.0134 $$

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