Solution #268

Z = Present Value R.V.

500 when Bill dies if John alive
1,000 when John dies if Bill dead

both 80
$i = 0\%$

Bill $\ell_x = 100(85 - x)$ $0 < x < 85$ De Moivre $\omega = 85$

John $\ell_x = 100(84 - x)$ $0 < x < 84$ De Moivre $\omega = 84$

$$\Rightarrow\Rightarrow \quad {}_tp_x = \frac{\omega - x - t}{\omega - x}$$

$$\mu_x = \frac{1}{\omega - x - t}$$

$E(Z) = ?$

In General $PVFB = \int_0^\infty v^t \, {}_tp_x \, \mu_{x+t} \, dt$

$\int v^t \, {}_tp_x^{(B)} \, \mu_{x+t}^{(B)} \, {}_tp_x^{(J)} \, dt$ integrate from 0 to 4

$\int v^t \, {}_tp_x^{(J)} \, \mu_{x+t}^{(J)} \, {}_tq_x^{(B)} \, dt$ integrate from 0 to 4

$$500 \int_0^4 \left(\frac{5-t}{5}\right)\left(\frac{1}{5-t}\right)\left(\frac{4-t}{4}\right) dt + 1000 \int_0^4 \left(\frac{4-t}{4}\right)\left(\frac{1}{4-t}\right)\left(\frac{t}{5}\right) dt$$

$v^t = 1$ because $i = 0\%$

$$\frac{500}{20}\int_0^4 (4-t)\,dt + \frac{1000}{20}\int_0^4 t\,dt$$

$$25\left[4t - \frac{t^2}{2}\Big|_0^4\right] + 50\left[\frac{t^2}{2}\Big|_0^4\right]$$

$$25(16-8) + 50(8)$$

$$200 + 400 = 600 \quad \boxed{A}$$

$\mu_{x+t}^{(B)}$ $\mu_{x+t}^{(J)}$ ${}_tp_x^{(B)}$