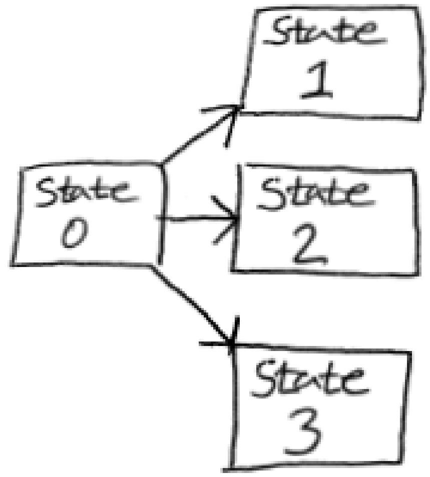
Solution #283

Given

$\mu_{x+t}^{01} = 0.3$

$\mu_{x+t}^{02} = 0.5$

$\mu_{x+t}^{03} = 0.7$



Calculate p_x^{02}

p_x^{02} = Probability of an x-year old in state 0 at time 0 being in state 2 at time 1

$$p_x^{02} = \int_0^1 {}_tp_x^{\overline{00}} \cdot \mu_{x+t}^{02} \cdot {}_{1-t}p_{x+t}^{\overline{22}} \, dt$$

$${}_tp_x^{\overline{00}} = \exp\left[-\int_0^t \mu_{x+s}^{0\bullet} \, ds\right]$$

$$*\mu_{x+s}^{0\bullet} = \mu_{x+s}^{01} + \mu_{x+s}^{02} + \mu_{x+s}^{03}$$

$$= 0.3 + 0.5 + 0.7$$

$$= 1.5$$

$${}_tp_x^{\overline{00}} = \exp(-1.5t)$$

$${}_{1-t}p_{x+t}^{\overline{22}} = 1 \rightarrow \text{It is not possible to exit state 2}$$

$$p_x^{02} = \int_0^1 {}_tp_x^{\overline{00}} \, \mu_{x+t}^{02} \, {}_{1-t}p_{x+t}^{\overline{22}} \, dt$$

$$= \int_0^1 e^{-1.5t} \cdot 0.5 \cdot 1 \, dt$$

$$= \left(\frac{1}{1.5}\right)(0.5) \cdot e^{-1.5t} \Big|_1^0$$

$$= \frac{1}{3}\left[1 - e^{-1.5}\right]$$

$$= 0.259 \approx 0.26$$

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