

## Solution 292

$$\text{Profit Margin} = \frac{\text{NPV}[\text{Profits}]}{\text{EPV}[\text{Future Premiums}]}$$

$Pr_t$  denotes the year- $t$  profit per policy in force at time  $t$

$(Pr_0, Pr_1, \dots, Pr_n)'$   $\rightarrow$  Profit vector denoting profit per survivor

$\Pi_t$  denotes the year- $t$  profit per policy issued

$(\Pi_0, \Pi_1, \dots, \Pi_n)'$   $\rightarrow$  Profit signature showing year-by-year profit expected for each policy issued

$$\Pi_0 = Pr_0$$

$$\Pi_t = {}_{t-1}P_x Pr_t \quad t > 0$$

$$P_{40} = 245/274$$

$$G = 1000$$

$${}_2P_{40} = 300/395$$

$$v = 1/1.12$$

$$\begin{aligned} \text{EPV}[\text{Future Premiums}] &= G [1 + vP_{40} + v^2{}_2P_{40}] \\ &= 1000 [1 + (245/274)v + (300/395)v^2] \\ &= 2403.821 \end{aligned}$$

$$\begin{aligned} \text{EPV}[\text{Profits}] &= \sum_{k=0}^n \pi_k v^k \\ &= -400 + 150v + 245v^2 + 300v^3 \\ &= 142.775 \end{aligned}$$

$$\begin{aligned} \text{Profit Margin} &= \text{NPV}(\text{Profits}) / \text{EPV}(\text{Premiums}) \\ &= 142.775 / 2403.821 \\ &= 5.94\% \approx 5.9\% \end{aligned}$$

(C)