

Question #195

Key: E

Let ${}_k P_0$ = Probability someone answers the first k problems correctly.

$${}_2 P_0 = (0.8)^2 = 0.64$$

$${}_4 P_0 = (0.8)^4 = 0.41$$

$${}_2 P_{0:0} = ({}_2 P_0)^2 = 0.64^2 = 0.41$$

$${}_4 P_{0:0} = (0.41)^2 = 0.168$$

$${}_2 P_{0:\overline{0}} = {}_2 P_0 + {}_2 P_0 - {}_2 P_{0:0} = 0.87$$

$${}_4 P_{0:\overline{0}} = 0.41 + 0.41 - 0.168 = 0.652$$

$$\begin{aligned}\text{Prob}(\text{second child loses in round 3 or 4}) &= {}_2 P_{0:\overline{0}} - {}_4 P_{0:\overline{0}} \\ &= 0.87 - 0.652 \\ &= 0.218\end{aligned}$$

$$\begin{aligned}\text{Prob}(\text{second loses in round 3 or 4} \mid \text{second loses after round 2}) &= \frac{{}_2 P_{0:\overline{0}} - {}_4 P_{0:\overline{0}}}{{}_2 P_{0:\overline{0}}} \\ &= \frac{0.218}{0.87} = 0.25\end{aligned}$$