

Exam P Problem 18

$h \sim$ height of tower

$X_1 \sim$ error, $N(0, .0056h)$

$X_2 \sim$ error₂ $N(0, .0044h)$

assume X_1 and X_2 independent

Probability of average error $\pm .005h$

Prob. average error $\Rightarrow y$

$\Pr(-.005h \leq y \leq .005h)$

$$y = \frac{X_1 + X_2}{2} \sim \text{Normal}$$

$$E(y) = E\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{2}[E(X_1) + E(X_2)] = \frac{1}{2}(0 + 0) = 0$$

$$\text{Var}(y) = \text{Var}\left(\frac{X_1 + X_2}{2}\right) = \left(\frac{1}{2}\right)^2 \text{Var}(X_1 + X_2)$$

Due to independence!

$$= \frac{1}{4} [\text{Var}(X_1) + \text{Var}(X_2)]$$

$$= \frac{1}{4} [(.0056h)^2 + (.0044h)^2]$$

$$= .000013h^2$$

$$\sigma = .003561h$$

$$y \sim N(0, .003561h)$$

$$\begin{aligned} \Pr(-.005h \leq y \leq .005h) &= \Pr(y \leq .005h) - \Pr(y \leq -.005h) \\ &= \Pr(y \leq .005h) - \Pr(y \geq .005h) \end{aligned}$$

$$\Pr(y \leq .005h) = \Pr\left(\frac{y - \mu}{\sigma} \leq \frac{.005h - \mu}{\sigma}\right)$$

$$= \Pr(Z \leq 1.404) \Rightarrow \text{table} \Rightarrow .9192$$

$$\begin{aligned}\Pr(-.005h \leq y \leq .005h) &= \Pr(y \leq .005h) - \Pr(y \geq .005h) \\ &= 2\Pr(y \leq .005h) - 1 \\ &= 2(.9192) - 1 \\ &= .8384 \quad \square\end{aligned}$$