

Problem 23

Teen	$p(T) = .08$	$p(C T) = .15$
Young Adult	$p(Y) = .16$	$p(C Y) = .08$
Midlife	$p(M) = .45$	$p(C M) = .04$
Senior	$p(S) = .31$	$p(C S) = .05$

Strategy:
Bayes' Theorem

$$p(A_k|B) = \frac{p(A_k) \cdot p(B|A_k)}{\sum_{i=1}^n p(A_i) \cdot p(B|A_i)} = \frac{p(A_k \cap B)}{p(B)}$$

Law of Total Probability

$$A_i = T, Y, M, S$$

$$B = C$$

$$A_k = Y$$

$$\begin{aligned} p(Y|C) &= \frac{p(Y) \cdot p(C|Y)}{p(T) \cdot p(C|T) + p(Y) \cdot p(C|Y) + p(M) \cdot p(C|M) + p(S) \cdot p(C|S)} \\ &= \frac{(.16)(.08)}{(.08)(.15) + (.16)(.08) + (.45)(.04) + (.31)(.05)} \\ &= \frac{.0128}{.012 + .0128 + .018 + .0155} \approx \underline{\underline{.22}} \end{aligned}$$

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