Problem 28

$p(X) = \frac{1}{5}$

$p(\text{ineffective vial} \mid X) = 10\%$

$p(\text{ineffective vial} \mid X') = 2\%$

X: the shipment is from company X

I: 1 out of 30 vials tested is ineffective

$p(I \mid X) = {}_1C_{30} \cdot (.1)(.9)^{29} = .141$

↳ $Y \sim \text{Binomial}(n=30, p=.1)$; $p(Y=1)$

↱ $Y \sim \text{Binomial}(n=30, p=.02)$; $p(Y=1)$

$p(I \mid X') = {}_1C_{30} \cdot (.02)(.98)^{29} = .334$

$$p(X \mid I) = \frac{p(I \mid X) \cdot p(X)}{p(I \mid X) \cdot p(X) + p(I \mid X') \cdot p(X')}$$

$$= \frac{.141\left(\frac{1}{5}\right)}{.141\left(\frac{1}{5}\right) + .334\left(\frac{4}{5}\right)} = .096$$

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