SOA Exam P 034 (Univariate Probability Dist.)

Let X = life time of a machine part

$f(x)$ is proportional to $(10+x)^{-2}$ on the interval $(0, 40)$

$$\Rightarrow f(x) = \begin{cases} a(10+x)^{-2} & 0 < x < 40 \\ 0 & o/w \end{cases}$$

Property of PDF:

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_0^{40} a(10+x)^{-2}\,dx = 1$$

$$\Rightarrow a\int_0^{40} (10+x)^{-2}\,dx = 1$$

$$\int_0^{40} (10+x)^{-2}\,dx = -(10+x)^{-1}\Big|_0^{40}$$

$$= \frac{1}{10} - \frac{1}{50}$$

$$= 0.08$$

$$a = \frac{1}{0.08}$$

$$= 12.5$$

$$f(x) = 12.5(10+x)^{-2} \quad 0 < x < 40$$

The probability that the lifetime is less than 6

$$= Pr(X < 6)$$

$$= \int_0^6 f(x)\,dx = \int_0^6 12.5(10+x)^{-2}\,dx$$

$$= 12.5\left[-(10+x)^{-1}\right]_0^6 = 12.5\left(\frac{1}{10} - \frac{1}{16}\right)$$

$$= 0.47$$

ANS: **C**