

Problem 42

X_A : total claim amount for Company A $\succ X_A, X_B$ independent

X_B : total claim amount for Company B

$$p(X_A = 0) = .6 \Rightarrow p(X_A > 0) = .4$$

$$p(X_B = 0) = .7 \Rightarrow p(X_B > 0) = .3$$

$$X_A | X_A > 0 \sim \text{Normal}(10,000, 2000^2)$$

$$X_B | X_B > 0 \sim \text{Normal}(9000, 2000^2)$$

$$p(X_B > X_A) = p(X_B > 0) \cdot p(X_A = 0) + p(X_B > 0) \cdot p(X_A > 0) \cdot \underbrace{p(X_B > X_A | X_B \cap X_A > 0)}_{p(X_B - X_A > 0 | X_B \cap X_A > 0)}$$

$$\begin{aligned} * X_A - X_B | X_B \cap X_A > 0 &\sim \text{Normal}(9000 - 10,000, 2000^2 + 2000^2) \\ &\sim \text{Normal}(-1000, 2 \cdot 2000^2) \end{aligned}$$

Y and Z normal, independent, random variables

$$E[cY + dZ] = c \cdot E[Y] + d \cdot E[Z]$$

$$V[cY + dZ] = c^2 \cdot V[Y] + d^2 \cdot V[Z]$$

$$\begin{aligned} c &= 1 \\ d &= -1 \end{aligned}$$

$$\begin{aligned} p(X_B - X_A > 0 | X_B \cap X_A > 0) &= p\left(Z > \frac{0 - [-1000]}{\sqrt{2 \cdot 2000^2}}\right) \\ &= p(Z > .35) \\ &= 1 - \Phi(.35) \\ &= 1 - .6368 = .3632 \end{aligned}$$

using Normal Table

$$\begin{aligned} p(X_B > X_A) &= .3(.6) + .3(.4)(.3632) \\ &= .18 + .043584 = \underline{\underline{.223584}} \end{aligned}$$

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