

Problem 43

A_i : one or more accidents occur in month i

A'_i : no accidents occur in month i

A_i s independent

$$p(A_i) = \frac{3}{5}$$

$$p(A'_i) = \frac{2}{5}$$

$p(\text{at least 4 months accident free before 4th month with accident}) = P(1) + P(2) + P(3) + P(4)$

Four Scenarios:

1. 4 consecutive accident free months
2. 4 accident free months and 1 with an accident
3. 4 accident free months and 2 with accidents
4. 4 accident free months and 3 with accidents

4th A'_i MUST occur
in the last month

$$1. p(4A'_i) = p(A'_i)^4 \quad \text{due to independence} \\ = \left(\frac{2}{5}\right)^4 = .0256$$

$$2. p(4A'_i \cap A_i | A'_5) = \binom{4}{1} \cdot p(A'_i)^3 \cdot p(A_i) \cdot p(A'_5) \\ = 4 \cdot \left(\frac{2}{5}\right)^4 \cdot \left(\frac{3}{5}\right) = .06144$$

$$3. p(4A'_i \cap 2A_i | A'_6) = \binom{5}{2} \cdot p(A'_i)^3 \cdot p(A_i)^2 \cdot p(A'_6) \\ = 10 \cdot \left(\frac{2}{5}\right)^4 \cdot \left(\frac{3}{5}\right)^2 = .09216$$

$$4. p(4A'_i \cap 3A_i | A'_7) = \binom{6}{3} \cdot p(A'_i)^3 \cdot p(A_i)^3 \cdot p(A'_7) \\ = 20 \cdot \left(\frac{2}{5}\right)^4 \cdot \left(\frac{3}{5}\right)^3 = .110592$$

$$.0256 + .06144 + .09216 + .110592 = .29$$

