Exam P Problem 52

Solution:

① Let us first determine K

Given that: * $\Pr(\text{a loss of amount } N) = \frac{K}{N}$

* K is a constant

* N = 1, 2, 3, 4, 5

$$\Pr(N=1) + \Pr(N=2) + \Pr(N=3) + \Pr(N=4) + \Pr(5) = 1$$

$$1 = K\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) = K\left(\frac{137}{60}\right) \Rightarrow K = \frac{60}{137}$$

② We know that: $\Pr(AB) = \Pr(AB \mid B)\Pr(B) = \Pr(AB \mid A)\Pr(A)$

$$\Pr(N=1,2,3,4,5) = \Pr(N=1,2,3,4,5 \cap \text{Insured suffers loss})$$

$$\Pr(N=n) = \Pr(N=n \mid \text{Insured suffers a loss})\Pr(\text{Insured suffers a loss})$$

$$= \frac{60}{137N}(0.05) = \frac{3}{137N} \qquad N = 1, \cdots, 5$$

③ Denote the Net Premium amount X, with deductible of 2

$$X = \begin{cases} 0 & \text{if } N \leq 2 \\ N-2 & \text{if } N > 2 \end{cases}$$

④ Then

$$E[X] = \Pr(N=n)\Pr(X=x) = \sum_{N=3}^{5}(N-2)\frac{3}{137N} = (1)\left(\frac{1}{137}\right) + 2\left(\frac{3}{137\times 4}\right) + 3\left[\frac{3}{137(5)}\right]$$

$$= .0314$$

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