

Exam P Problem 55

Given: pdf is proportional to $(1+x)^{-4}$

$$f(x) = k \cdot (1+x)^{-4} \quad 0 < x < \infty \quad k \text{ is a constant}$$

$$1 = \int_0^{\infty} \frac{k}{(1+x)^4} = -\frac{k}{3} (1+x)^{-3} \Big|_0^{\infty} = \frac{k}{3} \Rightarrow k=3$$

$$E[X] = \int_0^{\infty} x \cdot f(x) dx$$

$$= \int_0^{\infty} x \cdot \frac{3}{(1+x)^4} dx$$

substitute $u = 1+x$, $du = dx$

$$\Rightarrow E[X] = \int_1^{\infty} \frac{3(u-1)}{u^4} du = \int_0^{\infty} \frac{3u}{u^4} - \frac{3}{u^4} du$$

$$= 3 \int_0^{\infty} (u^{-3} - u^{-4}) du$$

$$= 3 \left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3} \right]_1^{\infty}$$

$$= 1/2$$

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