

Exam P 077 (Multivariate Prob Dist)

We are given $f(x, y) = \frac{x+y}{8}$ $0 \leq x \leq 2$, $0 \leq y \leq 2$

T = time until failure

We want to solve $\Pr(T \leq 1)$

$\Pr(T > 1) = \Pr(x > 1 \cap Y > 1)$ means both x & Y will work past $T=1$

$$\begin{aligned}\Pr(T > 1) &= \int_1^2 \int_1^2 \frac{x+y}{8} dy dx = \int_1^2 \left(\frac{xy}{8} + \frac{y^2}{16} \Big|_{y=1}^{y=2} \right) dx \\ &= \int_1^2 \left(\frac{x}{8} + \frac{3}{16} \right) dx = \frac{x^2}{16} + \frac{3x}{16} \Big|_1^2 = \frac{3}{8}\end{aligned}$$

Thus we have

$$\Pr(T \leq 1) = 1 - \Pr(T > 1) = 1 - \frac{3}{8} = \frac{5}{8} = \boxed{0.625}$$