

83.

Given: $X \sim N(3, 1)$ $E(X) = 3$ $\text{Var}(X) = 1$

→ we want n number of bulbs with the same distribution

$W = X_1 + X_2 + \dots + X_n$ w : time n # of bulbs last

pdf of W



(40 and .9772 are given)

$$P(W > 40) \geq 0.9772$$

$$\begin{aligned} E(W) &= E(X_1 + X_2 + \dots + X_n) \\ &= n E(X) \\ &= 3n \end{aligned}$$

$$E(A+B) = E(A) + E(B)$$

$$\begin{aligned} \text{Var}(W) &= \text{Var}(X_1 + X_2 + \dots + X_n) \\ &= \text{Var}(X) \cdot n \quad \leftarrow \text{since independent} \\ &= n \end{aligned}$$

$$P\left(\frac{W - E(W)}{\sqrt{\text{Var}(W)}} > \frac{40 - E(W)}{\sqrt{\text{Var}(W)}}\right) \geq 0.9772$$

$$P\left(\frac{W - 3n}{\sqrt{n}} > \frac{40 - 3n}{\sqrt{n}}\right) \geq 0.9772$$

$$2 = \frac{40 - 3n}{\sqrt{n}}$$

* 2 is the value from the z-table that corresponds with 0.9772

$$2\sqrt{n} = 40 - 3n$$

$$3n + 2\sqrt{n} - 40 = 0$$

$$x = \sqrt{n} \quad 3x^2 + 2x - 40 = 0$$

$$\frac{-2 \pm \sqrt{2^2 + 4(3)(-40)}}{2(3)} \Rightarrow$$

$$\sqrt{n} = 4$$

$$n = 16$$

B