

Problem 85

Givens P: premium for each policy
X: claim amount for one policy
N: number of policies sold

$$P = E(X) + 100$$
$$f(x) = \frac{1}{1000} \cdot e^{-\frac{x}{1000}}$$
$$N = 100$$

Asked: $p(\text{Total claim amount} \geq \text{Total premium})$

Strategy: Central Limit Theorem

$$X \sim \text{Exponential} (\theta = 1000)$$

$$E(X) = \theta = 1000$$

$$V(X) = \theta^2 = 1000^2 = 1,000,000$$

$$P = E(X) + 100 = 1100$$

$$\text{Total } P = N \cdot P = 110,000$$

S: Total claims

$$E(S) = N \cdot E(X) = 100,000$$

$$V(S) = N \cdot V(X) = (100)(1,000,000)$$

$$SD(S) = \sqrt{V(S)} = 10,000$$

$$\begin{aligned} p(S \geq 110,000) &= 1 - p(S \leq 110,000) \\ &= 1 - p\left(Z \leq \frac{110,000 - E(S)}{SD(S)}\right) \\ &= 1 - p(Z \leq 1) \\ &= 1 - .841 = \underline{\underline{.159}} \end{aligned}$$

Standardizing

from Normal table

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