

90. Two types of policies: Basic + Deluxe

Basic \sim Exp Dist. $w/\mu = 2$

$$f(t_1) = \frac{1}{2} e^{-\frac{t_1}{2}}$$

Deluxe \sim Exp. Dist. $w/\mu = 3$

$$f(t_2) = \frac{1}{3} e^{-\frac{t_2}{3}}$$

$T_1 + T_2$ are independent

$$f(t_1, t_2) = f(t_1)f(t_2) = \frac{1}{2} e^{-\frac{t_1}{2}} \cdot \frac{1}{3} e^{-\frac{t_2}{3}} \quad 0 < t_1, t_2 < \infty$$

$P(\text{next claim will be deluxe}) = P(T_2 < T_1)$

$$= \int_0^{\infty} \int_0^{t_1} f(t_1, t_2) dt_2 dt_1$$

* integral goes from 0 to t_1 , because $0 < t_2 < t_1 < \infty$

$$= \int_0^{\infty} \left(\frac{1}{2} e^{-\frac{t_1}{2}} e^{-\frac{t_2}{3}} \right)_0^{t_1} dt_1$$

$$= \left(-e^{-\frac{t_1}{2}} + \frac{3}{5} e^{-\frac{5}{6} t_1} \right) \Big|_0^{\infty} = \frac{2}{5}$$

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