

Problem 95

$$\underbrace{M_X(t) = M_Y(t)} = e^{\frac{t^2}{2}}$$
$$E(e^{t \cdot X}) = E(e^{t \cdot Y})$$

$$W = X + Y$$
$$Z = Y - X$$

$$M_{W,Z}(t_1, t_2) = E(e^{t_1 \cdot W + t_2 \cdot Z})$$
$$= E(e^{t_1 \cdot [X+Y] + t_2 \cdot [Y-X]})$$
$$= E(e^{[t_1 - t_2] \cdot X + [t_1 + t_2] \cdot Y})$$
$$= E(e^{[t_1 - t_2] \cdot X} \cdot e^{[t_1 + t_2] \cdot Y})$$
$$= E(e^{[t_1 - t_2] \cdot X}) \cdot E(e^{[t_1 + t_2] \cdot Y})$$

X and Y are independent

For independent variables, $E(X \cdot Y) = E(X) \cdot E(Y)$

$$= e^{\frac{1}{2}(t_1 - t_2)^2} \cdot e^{\frac{1}{2}(t_1 + t_2)^2}$$

$$= \underline{\underline{e^{t_1^2 + t_2^2}}}$$

\square