

Problem 105

$$f(x,y) = \frac{8}{3}xy \quad \begin{array}{l} 0 \leq x \leq 1 \\ x \leq y \leq 2x \end{array}$$

$$\begin{aligned} E(XY) &= \int_0^1 \int_x^{2x} \frac{8}{3}xy(xy) \cdot dy \cdot dx \\ &= \frac{8}{3} \int_0^1 \frac{x^2y^3}{3} \Big|_x^{2x} \cdot dx \\ &= \frac{8}{9} \int_0^1 7x^5 \cdot dx \\ &= \frac{8}{9} \left(\frac{7x^6}{6} \right) \Big|_0^1 = \frac{28}{27} \end{aligned}$$

$$\begin{aligned} E(X) &= \int_0^1 \int_x^{2x} \frac{8}{3}xy(x) \cdot dy \cdot dx \\ &= \frac{8}{3} \int_0^1 \frac{x^2y^2}{2} \Big|_x^{2x} \cdot dx \\ &= \frac{8}{6} \int_0^1 3x^4 \cdot dx \\ &= \frac{8}{6} \left(\frac{3x^5}{5} \right) \Big|_0^1 = \frac{4}{5} \end{aligned}$$

$$\begin{aligned} \text{cov}(X,Y) &= E(XY) - E(X) \cdot E(Y) \\ &= \frac{28}{27} - \frac{4}{5} \left(\frac{56}{45} \right) \\ &= .04 \\ &= \boxed{A} \end{aligned}$$

$$\begin{aligned} E(Y) &= \int_0^1 \int_x^{2x} \frac{8}{3}xy(y) \cdot dy \cdot dx \\ &= \frac{8}{3} \int_0^1 \frac{xy^3}{3} \Big|_x^{2x} \cdot dx \\ &= \frac{8}{9} \int_0^1 7x^4 \cdot dx \\ &= \frac{8}{9} \left(\frac{7x^5}{5} \right) \Big|_0^1 = \frac{56}{45} \end{aligned}$$