

106.

$$\text{Cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

$$X \sim U(a, b)$$

$$X \sim U(0, 12)$$

$$Y \sim U(0, x)$$

$$f_x(x) = \frac{1}{b-a}$$

$$f_x(x) = \frac{1}{12}$$

$$f_{y|x}(y|x) = \frac{1}{x}$$

$$E(X) = \frac{0+12}{2} = 6$$

$$E(X) = \frac{a+b}{2}$$

$$E(Y) = \int_0^{12} \int_0^x y \cdot f_{y|x}(x, y) dy dx$$

$$f_{y|x}(y|x) = \frac{f_{xy}(x, y)}{f_x(x)}$$

$$\Rightarrow f_{xy}(x, y) = f_{y|x}(y|x) \cdot f_x(x)$$

$$f_{xy}(x, y) = \left(\frac{1}{x}\right) \left(\frac{1}{12}\right)$$

$$E(Y) = \int_0^{12} \int_0^x y \cdot \left(\frac{1}{12x}\right) dy dx$$

$$= \int_0^{12} \frac{y^2}{2 \cdot 12x} \Big|_0^x dx$$

$$= \int_0^{12} x \cdot \frac{1}{24} dx$$

$$= x^2 \cdot \frac{1}{2 \cdot 24} \Big|_0^{12}$$

$$= 3$$

$$E(XY) = \int_0^{12} \int_0^x x \cdot y \cdot \frac{1}{12} \cdot \frac{1}{x} dy dx$$

$$= \int_0^{12} \int_0^x \frac{y}{12} dy dx$$

$$= \int_0^{12} \frac{y^2}{2 \cdot 12} \Big|_0^x dx$$

$$= \int_0^{12} \frac{x^2}{24} dx$$

$$= x^3 \cdot \frac{1}{3 \cdot 24} \Big|_0^{12}$$

$$= \underline{\underline{24}}$$

$$\text{Cov}(XY) = E(XY) - E(X) \cdot E(Y)$$

$$= 24 - 6 \cdot 3$$

$$= 6$$

$$\boxed{6}$$